Quantum interference in nanometric devices: Ballistic transport across arrays of T-shaped quantum wires

AIP Applied Physics Letters

Quantum interference in nanometric devices: Ballistic transport across arrays of T-shaped quantum wires

Guido Goldoni, Fausto Rossi, and Elisa Molinari

Citation: Appl. Phys. Lett. **71**, 1519 (1997); doi: 10.1063/1.119954
View online: http://dx.doi.org/10.1063/1.119954
View Table of Contents: http://apl.aip.org/resource/1/APPLAB/v71/i11
Published by the American Institute of Physics.

Quantum interference in nanometric devices: Ballistic transport across arrays of T-shaped quantum wires

Guido Goldoni,[a)] Fausto Rossi, and Elisa Molinari
Istituto Nazionale per la Fisica della Materia (INFM) and Dipartimento di Fisica, Università di Modena, I-41100 Modena, Italy

(Received 20 May 1997; accepted for publication 16 July 1997)

We propose that recently realized T-shaped semiconductor quantum wires (T wires) could be exploited as three-terminal quantum interference devices. T wires are formed by intersecting two quantum wells (QWs). By use of a scattering matrix approach and Landauer–Büttiker theory, we calculate the conductance for ballistic transport in the parent QWs and across the wire region as a function of the injection energy. We show that different conductance profiles can be selected by tailoring the widths of the QWs and/or by combining more wires on the scale of the Fermi wavelength. Finally, we discuss the possibility of obtaining spin-dependent conductance of ballistic holes in the same structures. © *1997 American Institute of Physics.* [S0003-6951(97)04237-X]

T-shaped quantum wires (T wires) are semiconductor structures where quasi-one-dimensional (q1D) confinement is achieved at the intersection of two quantum wells (QWs).[1] T wires are obtained by first growing a GaAs/$Al_xGa_{1-x}As$ superlattice (labeled QW1) on a (001) substrate; after cleavage, a GaAs QW (labeled QW2) is grown over the exposed (110) surface, resulting in an array of T-shaped regions where electron and hole wave functions can be confined on a scale of a few (5–10) nm. Up to now, intensive investigation of these structures has focused on optical properties, and it has demonstrated strong one-dimensional quantum confinement of the lowest excitonic transitions[2] as well as evidence of laser emission.[3] Transport experiments along the wires were first obtained only very recently.[4]

Different from wires obtained by other techniques, such as V-shaped or deep-etched wires,[5] the section of a T wire has an open geometry. Therefore, in addition to transport along the quantum wire in q1D bound states, parallel transport in the constituent QWs and across the wire region becomes possible if the two-dimensional (2D) continuum is contacted (for example through the overgrown layer). In addition to q1D bound states, which fall below the 2D continuum edge of the parent QWs, q1D resonant states exist within the 2D continuum.[6] The injected carriers that travel ballistically over the wire region (nm scale) will show a strong energy dependent transmission as a consequence of quantum interference effects induced either by resonant q1D states or by the interplay between the propagating modes of the parent QWs.

In semiconductors, quantum interference effects are normally achieved in channels defined by gating an underlying 2D high mobility electron gas with electrostatic potentials. Structures of this type with T-shaped geometries have been proposed to achieve device functions;[7] in this case, the conductance along a channel can be controlled by modulating the length of a lateral, closed arm (stub).[8,9] Instead, in the present T wires the lateral arm (QW1) is open, and the conductance is controlled by modulating the chemical potential (i.e., the injection energy). As we will show, different shapes of the conductance as a function of energy can be selected by tailoring the widths of the QWs and/or by combining more wires. In the proposed experiment with T wires, the interference patterns should be stable in a much larger temperature range than in previously proposed structures, due to the nm size confinement and the large intersubband splittings in the parent QWs (of the order of 0.1 eV); furthermore, the confinement is provided by high-quality interfaces, as demonstrated by the small excitonic linedwidths.[2]

In the following we calculate the ballistic conductance for parallel transport in the QWs through the T-wire intersection. Assuming perfect barrier confinement in the parent QWs, the calculation of the scattering matrix[10] presents no conceptual difficulty. We divide the sample into four regions (see inset of Fig. 1); in each region the wave function of energy E is written as a linear combination of the propagating and evanescent modes of the corresponding QW. Indicating with $E_{1,n}$ and $f_{1,n}(x)$ the subband energies and envelope functions of QW1, and with $E_{2,n}$ and $f_{2,n}(y)$ those of QW2, we have, for zero in-wire momentum,

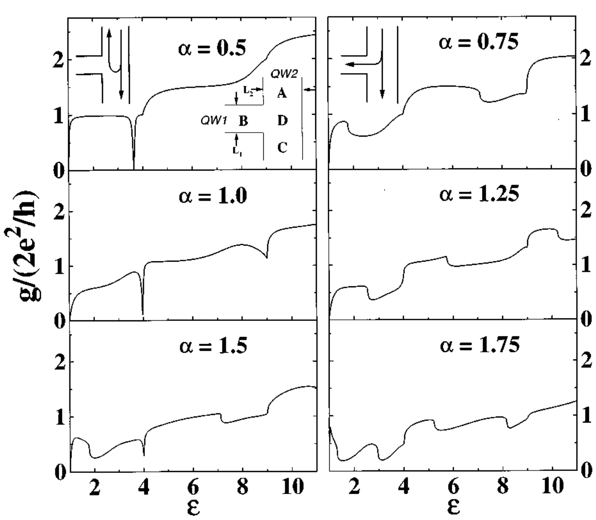

FIG. 1. Two-terminal conductance vs injection energy for selected values of α, according to the labels. The relevant geometric parameters are defined in the inset.

[a)]Electronic mail: goldoni@unimo.it

$$\psi_A=\sum_n f_{2,n}(y)(a_n^> e^{i\xi_n x}+a_n^< e^{-i\xi_n x}), \tag{1a}$$

$$\psi_B=\sum_n f_{1,n}(x)(b_n^< e^{i\eta_n y}+b_n^> e^{-i\eta_n y}), \tag{1b}$$

$$\psi_C=\sum_n f_{2,n}(y)(c_n^< e^{i\xi_n x}+c_n^> e^{-i\xi_n x}), \tag{1c}$$

$$\psi_D=\sum_n f_{2,n}(y)(d_n^+ e^{i\xi_n x}+d_n^- e^{-i\xi_n x}) + \sum_n f_{1,n}(x)e_n[e^{i\eta_n y}-e^{-i\eta_n y}), \tag{1d}$$

where the wave vectors ξ_n, η_n are given by $\xi_n^2=(\epsilon-n^2)\times(\pi/L_2)^2$ and $\eta_n^2=(\epsilon-n^2/\alpha^2)(\pi/L_2)^2$. Here, $\epsilon=E/E_{2,1}$ is the energy in units of the lowest mode of QW2, $E_{2,1}=\hbar^2\pi^2/2mL_2^2$, and $\alpha=L_1/L_2$. The two equations obtained at each interface by matching both ψ and its normal derivative are projected over the nth mode (i.e., multiplied by the appropriate sine or cosine function and integrated over the interface) and finally summed and subtracted to obtain two new equations, relating either to the incoming or the outgoing wave coefficient through that interface to the coefficients of the inside region D. Including N modes[11] in the sums in (1a)–(1d), and defining the vector $\mathbf{a}^>=(a_1^>,a_2^>,\ldots)$ and analogously for the other coefficients, we obtain a set of linear equations of the form

$$\mathbf{a}^{\gtrless}=\mathbf{d}^{\pm}+\mathbf{P}^{\pm}\cdot\mathbf{e}, \tag{2a}$$

$$\mathbf{b}^{\lessgtr}=\mathbf{V}^{\pm}\cdot\mathbf{d}^{+}+\mathbf{W}^{\pm}\cdot\mathbf{d}^{-}+\mathbf{e}, \tag{2b}$$

$$\mathbf{c}^{\lessgtr}=\mathbf{d}^{\pm}+\mathbf{Q}^{\pm}\cdot\mathbf{e}, \tag{2c}$$

where the eight $N\times N$ matrices $\mathbf{P}^{\pm}$, $\mathbf{Q}^{\pm}$, $\mathbf{V}^{\pm}$, and $\mathbf{W}^{\pm}$ ensue from the matching conditions and depend on the geometrical parameters and on the energy. By defining the incoming and outgoing states $|\mathrm{in}\rangle=(\mathbf{a}^>,\mathbf{b}^>,\mathbf{c}^>)$ and $|\mathrm{out}\rangle=(\mathbf{a}^<,\mathbf{b}^<,\mathbf{c}^<)$, and appropriate $3N\times 3N$ matrices $\mathbf{F}$, $\mathbf{G}$ in terms of the eight matrices above, Eqs. (2a)–(2c) can be rewritten as

$$|\mathrm{out}\rangle=\mathbf{F}(\mathbf{e},\mathbf{d}^+,\mathbf{d}^-)^T, \quad |\mathrm{in}\rangle=\mathbf{G}(\mathbf{e},\mathbf{d}^+,\mathbf{d}^-)^T. \tag{3}$$

Combining Eqs. (3) we finally get

$$|\mathrm{out}\rangle=\mathbf{F}\cdot\mathbf{G}^{-1}|\mathrm{in}\rangle=\mathbf{S}|\mathrm{in}\rangle. \tag{4}$$

Equation (4) defines the scattering matrix $\mathbf{S}$, which at the same time gives the bound state ($\epsilon<1$),[12] satisfying the equation $\det(\mathbf{S}^{-1})=0$, and the scattering states ($\epsilon>1$).

Since the scattering matrix is a property of the potential at a given energy ϵ, it allows one to calculate all the transmission coefficients, say, from mode n in arm A to mode m in arm C, by the same matrix, choosing the appropriate state $|\mathrm{in}\rangle$; to keep on with the example of A→C transmission, the transmission coefficient is $t_{n,m}^{\mathrm{AC}}=|c_n^</a_m^>|^2\xi_n/\xi_m$. In the following we shall concentrate on straight (i.e., A→C) transmission along QW2. We consider a configuration in which arm B is kept at the same potential of arm C ($V_\mathrm{B}=V_\mathrm{C}$). Therefore, no carrier is injected into the structure through arm B, and $\mathbf{b}^>=0$.[13] Using Eq. (2b), we can eliminate the $\mathbf{e}$ coefficients from the equations and we can rewrite Eqs. (2a) and (2c) as

$$\begin{pmatrix}\mathbf{c}^<\\ \mathbf{c}^>\end{pmatrix}=\mathbf{T}\begin{pmatrix}\mathbf{a}^>\\ \mathbf{a}^<\end{pmatrix}, \tag{5}$$

where $\mathbf{T}$ is the $2N\times 2N$ A→C transfer matrix. Note that, if $V_\mathrm{B}>V_\mathrm{C}$, a case which we shall not investigate here, a certain amount of charge would be incoherently injected into the system through arm B, and the transfer matrix would then contain an incoherent part which, in a three-terminal device, was discussed by Büttiker.[14] In Ref. 14 the ratio between the coherent and incoherent parts of a two-terminal conductance is modulated through the tunneling probability into a third, randomizing terminal. The present T-shaped wires with $V_\mathrm{B}>V_\mathrm{C}$ might in fact be a system to implement such an experiment, with the tunneling probability into arm B being adjusted through the injection energy.

Going back to the $V_\mathrm{B}=V_\mathrm{C}$ case, the two-terminal Landauer–Büttiker (LB) conductance[15] is

$$g=\frac{2e^2}{h}\sum_{i,j} t_{i,j}^{\mathrm{AC}}. \tag{6}$$

In Fig. 1 we show the dimensionless conductance $g/2e^2/h$ as a function of the energy ϵ and also for selected values of the parameter α. We recognize two types of behavior: for samples in which the width of side arm, QW1, matches an integer number of semiperiods of the incoming wave ($\alpha=0.5$, 1, 1.5, left panels) there are strong reflection resonances[16] at the energies of resonant q1D states localized at the intersection; when these states appear, their energy is at or slightly below the onset of a new propagating state along QW2 which, in the present units, is at $n^2=1,4,9,\ldots$. When the matching condition is not fulfilled ($\alpha=0.75$, 1.25, 1.75, right panels), the conductance shows, on top of a regular increase, square-wave behavior, with sudden drops and rises when new propagating channels open in QW1 or QW2, i.e., the current coming from arm A flows into the side arm B or into the straight arm C, depending on the energy.

If successive wires in an array are at a distance larger than the coherence length, incoherent scattering will redistribute carriers homogeneously among the propagating modes; therefore, the conductance of N incoherently coupled wires is G^N, apart from possible broadening due to fluctuations in the QW widths on the monolayer scale. This would not wash out the interference patterns completely, however, as long as the intersubband splittings are large. Conversely, the LB conductance of a single wire can be changed by coupling more wires on the scale of the Fermi wavelength; this possibility is a distinct advantage of structures grown by epitaxy. As an example, we consider two coupled T wires with a barrier of width L_d between two QW1s (see inset in Fig. 2). The transmission coefficient of the whole structure can be calculated easily since the total $\mathbf{T}$ matrix is the product of the $\mathbf{T}$ matrices of the isolated wires. In Fig. 2(a) we compare, for the case $\alpha=0.5$, the conductance of a single wire with the conductance of two coupled wires for $L_b=L_1$ and $L_b=2L_1$. In the first case, the conductance shows a double resonance, which is a fingerprint of the bonding and anti-

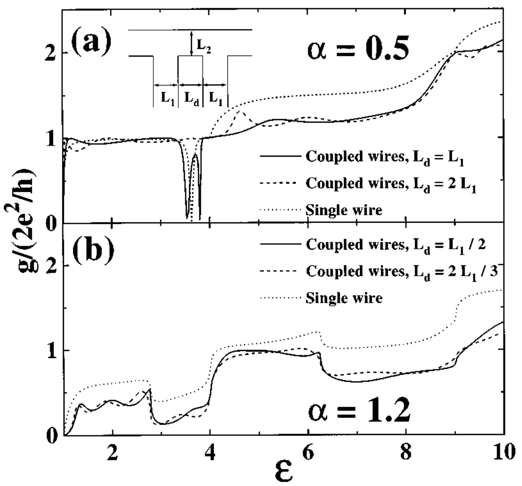

FIG. 2. Two-terminal conductance vs injection energy for a single wire and for two coupled wires for (a) $\alpha=0.5$ and (b) $\alpha=1.2$ and for selected values of L_b. The relevant geometric parameters are defined in the inset.

bonding combinations of the resonant q1D state of the isolated wires. But in the second case, the resonance is completely suppressed. In Fig. 2(b) we compare, for the case $\alpha=1.2$, the single and coupled wire conductance with $L_b=L_1/2$ and $L_b=2L_1/3$. The coupled wire case shows sharper modulations in comparison to the isolated wire case.

Finally we consider the possibility of transmitting holes, instead of conduction electrons, through a T wire. The valence subbands of a T wire are strongly spin split at finite in-wire wave vectors.[6] This can be understood in the following way. If the parent QWs of a T wire were isolated, the spin-degenerate valence subbands would cross at some finite in-wire wave vector because of the different effective masses for (001)- and (110)-grown wells. In each QW valence states can be characterized by the component of the total angular momentum $J=3/2$, the J quantization axis being along the growth direction.[17] Therefore, a state with a well defined component J_z, say, $J_z=3/2$ in one QW, is a mixture of $J_z=\pm 3/2, \pm 1/2$ states in the other QW; as a consequence, the strong spin-orbit coupling of valence states, by coupling heavy hole ($J_z=\pm 3/2$) and light hole ($J_z=\pm 1/2$) states, removes the degeneracy and results in a large spin splitting.[6] Therefore, if holes cross the wire region having a finite component of the wave vector along the wire axis, the transmitted current at selected energies could be strongly spin polarized.

In summary, we have proposed a nanostructure interference device based on cleaved-edge-overgrown T-shaped quantum wires, and have shown that its conductance profile can be tailored by chosing appropriate widths of the constituent QWs and of the barriers between adjacent structures. Possible applications as spin-selective devices for holes were also proposed, but they will require further theoretical and experimental investigation.

[1] L. N. Pfeiffer, K. W. West, H. L. Stormer, J. P. Eisentstein, K. W. Baldwin, D. Gershoni, and J. Spector, Appl. Phys. Lett. **56**, 1697 (1990); D. Gershoni, J. S. Weiner, S. N. G. Chu, G. A. Baraff, J. M. Vandenberg, L. N. Pfeiffer, K. W. West, R. A. Logan, and T. Tanbun-Ek, Phys. Rev. Lett. **65**, 1631 (1990).

[2] T. Someya, H. Akiyama, and H. Sakaki, Phys. Rev. Lett. **76**, 2965 (1996); H. Gislason, W. Langbein, and J. M. Hvam, Appl. Phys. Lett. **69**, 3248 (1996); see also F. Rossi, G. Goldoni, and E. Molinari, Phys. Rev. Lett. **78**, 3527 (1997), and references therein.

[3] W. Wegscheider, L. P. Pfeiffer, M. M. Digman, A. Pinczuk, K. E. West, S. L. McCall, and R. Hull, Phys. Rev. Lett. **71**, 4071 (1993); W. Wegscheider, L. P. Pfeiffer, and K. West, Appl. Phys. Lett. **65**, 2510 (1994).

[4] A. Yacoby, H. L. Stormer, N. S. Wingreen, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **77**, 4612 (1996).

[5] For a review, see for example, R. Cingolani and R. Rinaldi, Riv. Nuovo Cimento **16**, 1 (1993).

[6] G. Goldoni, F. Rossi, E. Molinari, and A. Fasolino, Phys. Rev. B **55**, 7110 (1997).

[7] F. Sols, M. Macucci, U. Ravaioli, and K. Hess, Appl. Phys. Lett. **54**, 350 (1989).

[8] K. Aihara, M. Yamamoto, and T. Mizutani, Appl. Phys. Lett. **63**, 3595 (1993); P. Debray, R. Akis, P. Vasilopoulos, and J. Blanchet, *ibid.* **66**, 3137 (1995).

[9] Note the different dimensionality of the devices obtained by gating a 2D electron gas with respect to the device proposed here: In Refs. 7 and 8 the lateral arms of the T are q1D channels, therefore termed ''wires.'' In T wires obtained by cleaved-edge overgrowth, the lateral arms of the T are sections of 2D systems, and the term ''wire'' refers to the q1D nature of the states localized at the intersection of such QWs.

[10] See, for example, E. Merzbacher, *Quantum Mechanics* (Wiley International, New York, 1970), p. 93.

[11] In principle, Eqs. (2a)–(2c) can only be satisfied by an infinite number of modes, which means that we have to check the convergence of our results with respect to N. In practice, since the energy of each mode increases as n^2 in an infinite well, very few modes are necessary.

[12] For a wire with $L_1=L_2$, the bound state is always at $\epsilon=0.807$; when the parent QWs are different, $0.807<\epsilon<1$.

[13] If arm B were terminated by a perfectly reflecting wall at a distance L, as in Refs. 7 and 8, we should set $b_n^> = -b_n^< \exp(2i\eta_n L)$ to account for the phase accumulated going along arm B.

[14] M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988).

[15] R. Landauer, IBM J. Res. Dev. **1**, 233 (1957); Philos. Mag. **21**, 863 (1970); M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).

[16] Z. Shao, W. Porod, and C. S. Lent, Phys. Rev. B **49**, 7453 (1994).

[17] G. Bastard, *Wave Mechanics Applied to Semiconductor Heterostructures* (Les Éditions des Physique, Les Ulis, France, 1988), p. 101.